Rally-Based Performance Evaluation Model for Highly Competitive Volleyball

Diego Luis González * [1], Ivan Gonzalez-Cabrera[2,3], and Diego Darío Herrera[3]

[1]Department of Physics, Universidad del Valle, A.A. 25360, Cali, Colombia
[2]Department of Psychology, University of Konstanz, 78457 Konstanz, Germany
[3]Rotation Technologies, 305-4476 Montreal, Canada

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Abstract

We propose a simple stochastic model to evaluate the effect of different complexes' performance on the probability of winning a rally. The model uses as input the probabilities of success and failure in various complexes, which can be extracted from standard match reports. Our model reproduces well-established results; for example, we found that a team that starts the rally with a serve is more likely to obtain a point in the phase of complex k_2 than in the phases associated with complexes k_0 and k_1. Conversely, if a team starts the rally receiving, it is more likely to win the rally in complex k_1. The proposed model also provides a new approach to quantify a team's performance in a rally and diagnose performance issues in different complexes. As a case study, we analyze the performance of a top South American team in the CSV Men's Tokyo Volleyball Qualification 2020. Although our model can be applied to various individual actions, our performance analysis focuses on one pivotal game action: the serve. It is found that only power jump serves that decrease the attacking efficiency in k_1 of the rival team have the potential to be more effective than jump float serves. The proposed model makes it possible to determine when one player's serve is more effective than another's, not only based on the number of direct points scored for or against but also on their influence on the probability of winning the rally.

*Corresponding Author: Diego Luis González (diego.luis.gonzalez@correounivalle.edu.co)
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1 Introduction

Statistical analysis of the capabilities of a team and its players is an essential part of the performance analysis in highly competitive volleyball [1, 2, 3, 4, 5? , 6, 7]. The results of these studies are used by coaches for the tactical planning of matches and training sessions. Some of the metrics used to evaluate the performance of a team, a player, or a group of players are efficacy and efficiency [8]. Efficacy, ξ, is a measure of whether predetermined goals are achieved regardless of cost. It is defined as the ratio between the positive actions A_+ and the total number of actions A_T, $\xi = A_+/A_T$. In contrast, efficiency, η, takes into account not only the result obtained from a particular game action but also its cost. In this case, the ratio explicitly includes the number of negative actions A_-, $\eta = (A_+ - A_-)/A_T$.

To illustrate the difference, consider the case of attack performance. Suppose a player, in a total of ten actions (spikes), scored four points and made four attack errors (e.g., the ball fails to go over the net, goes out of bounds, etc.). In the remaining two actions, the rally continues. In this scenario, we have $A_T = 10$, $A_+ = 4$, and $A_- = 4$, which implies that the player's efficacy is $\xi = 2/5$, whereas their efficiency is zero, $\eta = 0$. These two metrics differ; while ξ is always positive, η can be negative.

More elaborate methods divide the outcome of the action (or group of actions) into more than two categories, depending on the result obtained, assigning a weight to each category. For example, an action can be categorized as positive, negative, or neutral, with their respective weights c_+, c_-, and $c_=$. The values of the weights and the categorization items are arbitrary since they are defined by the technical team and depend on the action or set of actions considered. If the number of positive, negative, and neutral results obtained for a given action are A_+, A_-, and $A_=$, respectively, then the score, $\mathcal{C}$, of that action is given by

$$\mathcal{C} = \frac{c_+A_+ + c_-A_- + c_=A_=}{c_+A_T}. \tag{1}$$

Let us assume that in the case of the attack mentioned earlier, the coach assigns $c_+ = 2$, $c_- = -1$, and $c_= = 1$. In this example, the neutral actions correspond to $A_= = 2$. Thus, the score for the game action in consideration is $\mathcal{C} = 3/10$. As with efficacy and efficiency, the maximum value that $\mathcal{C}$ can have is 1. Therefore, the higher ξ, η, or $\mathcal{C}$, the better the performance associated with the action.

Any of these methods can be applied to individual actions such as serving, passing, setting, etc., as well as to a set of actions like those carried out in the different cycles of the rally. However, and more importantly for the purpose of this paper, another way to evaluate the performance of a given action or set of actions is through its impact on the probability of winning a rally. For instance, let us consider the particular case of serving. The result of this specific action can be divided into four categories: *positive* when a direct point (ace) is scored, *negative* when a point is conceded to the opponent (e.g., the serve does not go over the net, the ball goes out of bounds), *useful* when a point is not scored but the opponent's attack options are reduced, and *neutral* when the serve is completely controlled by the opponent. Depending on their skills and type of serve, a player has a given probability of scoring a point, conceding a point, or making a useful or neutral serve. Thus, a player's service performance can be evaluated based on its effect on the probability that the player's team wins the rally with the player serving.

A convenient way to categorize the different actions involved in a rally is through the concept of a complex. As described in Refs. [9, 10], the different complexes are defined according to the actions of the game they involve. Each rally begins with one of the two teams serving. This action is called complex k_0. The team that receives the opponent's serve is in a game situation called complex k_1, which includes the actions of receiving, setting, and attacking. While a team executes complex k_1, its opponent is in a game situation called k_2, which includes the actions of blocking, defending, setting, and counterattacking. Finally, we have complex k_3, which includes the same actions as complex k_2.

The complexes k_1, k_2, and k_3 involve more game actions than complex k_0. From this point of view, the simplest complex is k_0 since it only involves one action: the serve.

Building upon previous Markovian models that have been used to calculate the winning probability of a set in a volleyball match [10], the proposed model helps to evaluate the effect of players' actions during the different complexes of a rally on the probability of winning the rally. We believe that this approach is valuable not only for coaches to improve the performance evaluations of their players but also as a tool to design more efficient game strategies and training routines. The proposed model is more detailed than those described before, allowing for the diagnosis of performance issues in the various actions that make up a rally.

In particular, we focus on the impact of service performance on a team's probability of winning a rally given that a particular player is serving. However, it is important to note that our model can diagnose possible performance issues in other actions, such as receiving, blocking, and attacking. The proposed model takes into account different sets of actions involved in a rally and classifies service actions as positive, negative, neutral, or useful. Unlike previous modeling work, our model not only evaluates an action or set of actions but also quantifies its effect on the rest of the actions required to win a rally. Moreover, our model is not purely descriptive; it also quantifies how an improvement in one or more actions would change the probability of winning a rally. In this approach, a server's performance is better than another's if, under the same conditions, it increases the probability of winning a rally.

Finally, our focus on serve performance is not arbitrary. The serve is a crucial game action because, to score n consecutive points, it is necessary to win $n-1$ rallies while serving. In addition, to win a set, it is necessary to have at least a two-point lead. Another important aspect is that, in volleyball, it is more likely to score a point if you start the rally receiving than serving. In this way, the serving team is disadvantaged and must use the serve to reduce the rival's chances of success. The current data suggests that men tend to prefer the power jump serve in volleyball, though this preference might differ among women [11]. This type of serve is commonly used in high-level volleyball [12] and is thought to have a negative impact on reception performance, which in turn affects the setting zone [13]. Volleyball coaches often emphasize the application of pressure with the serve, even though this increases the risk of making errors. Since the benefit of applying pressure may not always outweigh the increased risk of service errors, one of the key applications of our model is to help volleyball teams determine when this risk is worth it, based on the overall impact of the players' serve on rally success.

The rest of the paper is divided as follows. In Sec. 2, we describe our model and present the typical values of the input probabilities reported in the literature. In Sec. 3, the model is used to reproduce well-known results, such as the probability of winning a rally given that the team starts serving or receiving. Then, we discuss some theoretical implications of the model, including the importance of the serves that do not score a direct point but decrease the attack variants of the rival. Furthermore, we use our model to analyze the performance of a top South American team and show how the model can be used to diagnose performance issues in actions other than serving. Our analysis includes some specific recommendations to improve the team's performance significantly. Finally, in Sec. 4, we provide some final remarks.

2 Model Description

Sports like volleyball, tennis, and racquetball are sequential and cyclical in the following sense. A set is divided into independent rallies. Each rally involves the repetition of several game actions until one of the two teams scores and the rally ends. The team that won the previous rally starts the next rally serving. This characteristic facilitates the mathematical modeling of these sports since it allows them to be described, as a first approximation, by means of Markov chains [14, 15**?** , 16, 10, 17, 18, 19, 20, 21, 22, 23]. Like ours, however, these models are sport-specific. The application of Markovian methods to one sport cannot be straightforwardly extrapolated to the analysis of another, not even between beach and indoor volleyball.

As explained in the previous section, the complexes are defined according to the game actions they involve. Each complex has a particular objective. Complex k_1 seeks to neutralize the rival's serve and set up an attack to win the rally or, if this is not possible, to decrease the rival team's probability of success in complex k_2. Complexes k_2 and k_3 are intended to contain the rival team's attack and organize a counterattack. Complex k_0, in turn, seeks to score a direct point while serving or, at least, to reduce the opponent's attacking options in complex k_1. Thus, each complex can be characterized by the following probabilities:

- q_{k_i} (r_{k_i}) the probability that team A (B) scores a point when executing the k_i complex.
- q'_{k_i} (r'_{k_i}) the probability that team A (B) loss the point when executing the k_i complex.

For simplicity, we make two assumptions. First, we assume that complex k_3 is equivalent to k_2, which is reasonable because those complexes involve essentially the same game actions. Second, we consider that the probabilities q_{k_i}, q'_{k_i}, r_{k_i}, and r'_{k_i} do not change throughout the rally. Therefore, we do not take into account that long rallies favor the team with the best physical preparation and neglect psychological factors [24, 25]. Since only three types of complexes are considered, we have that $i = 0, 1, 2$. For example, according to the previous definitions, q_{k_0} is the probability of scoring a direct serve point, and q'_{k_0} is the probability of making an error while serving (e.g., the serve fails to go over the net, goes out of bounds, the server commits a foot violation, etc.). Additionally, by definition, $Q_{k_i} = 1 - q_{k_i} - q'_{k_i}$ ($R_{k_i} = 1 - r_{k_i} - r'_{k_i}$) is the probability that, once team A (B) executes complex k_i, the rally continues. The probability that given that the rally continues, the ball remains in possession of the team that executed the complex is s, while the probability that the opposing team takes possession of the ball is c. An example of the former situation occurs when the attack of a team in k_1 or k_2 impacts the rival block in such a way that the attacking team is back in possession of the ball (which occurs with probability s). In this situation, the team that initially attacked can set up a new attack. However, if after the initial attack, the ball is in possession of the defending team (which occurs with probability c), this team can counterattack. In high-performance volleyball, c is about four times larger than s, so we assume that $c = 0.8$ and $s = 0.2$ are reasonable values for these probabilities.

In order to estimate the probabilities defined above, the model needs data for the different complexes that are taken into account. In the case of k_0, three different types of serves are considered: power jump serve (JS), jump float serve (JFS), and static floating serve (FS). Table 1 shows the statistics for the three types of serves reported in Ref. [26]. The data were taken from 4552 serves in 28 matches during the 2008-2009 regular season of the Italian volleyball male Top League and differentiate four different results:

- Error: point for the opposing team.
- Neutral: the serve is controlled by the opposing team in such a way that its reception allows it to set the ball to all possible attackers.
- Useful: the serve is controlled with difficulty by the opposing team in such a way that its reception does not allow all possible attack variants.
- Positive: direct point for the team that serves.

Table 1: Evaluation of serve outcomes [26].

Serve Type	Negative	Neutral	Useful	Positive	Total
JS	690	2007	253	231	3181
JFS	80	1063	55	25	1223
FS	2	142	4	0	148

Table 2: Probabilities associated with complex k_0. The probability Q_{k_0} includes useful and neutral actions.

Serve	q_{k_0}	q'_{k_0}	Q_{k_0}
JS	0.07	0.22	0.71
JFS	0.02	0.07	0.91
FS	0.00	0.01	0.98

It is important to bear in mind that the classification of serve types proposed in Ref. [26] is arbitrary. Other characterizations are possible. However, the four-level description presented is the simplest one that allows us to describe the most relevant serve outcomes. In addition, more detailed characterizations require larger data sets to determine the probabilities of each category.

From these data, it is possible to calculate the probabilities associated with complex k_0: q_{k_0}, q'_{k_0}, and Q_{k_0}, which are shown in Table 2. According to these data, in elite volleyball, the probability of making a direct serve point, regardless of the type of serve, is approximately 0.06, while the probability associated with giving the point to the opponent by a serve error is close to 0.17 [26]. Therefore, if the type of serve is not discriminated, $q_{k_0} \approx 0.06$ and $q'_{k_0} \approx 0.17$. However, it is important to emphasize that these probabilities strongly depend on the type of serve. For the JS, the probability of error is greater than 0.21, while for the JFS and FS, the associated probabilities are approximately 0.06 and 0.01, respectively. In addition, the probability of making a direct serve point is close to 0.07 for the JS, 0.02 for the JFS, and close to zero for the FS. The probability that the rally continues after the serve is given by the sum of the neutral and useful serves reported in Table 1. For the JS, JFS, and FS, we have that Q_{k_0} is approximately 0.71, 0.91, and 0.98, respectively. The values of the probabilities used as input in the model for complex k_0 are summarized in Table 2.

On the other hand, following the results obtained in Ref. [27], the probability that a team scores a point while attacking in complex k_1 is close to $q_{k_1} \approx 0.5$, and the probability of committing an error in that complex is $q'_{k_1} \approx 0.16$. For complex k_2, the analogous probabilities are $q_{k_2} \approx 0.42$ and $q'_{k_2} \approx 0.18$. These values do not take into account that the values of q_{k_1} and q'_{k_1} for useful serves are usually different from those for neutral serves.

Based on Fig. 1, the probability P_A^s that team A wins a rally given that it starts serving can be determined in terms of the input probabilities of the model. To calculate P_A^s, it is important to note that once complex k_1 is executed by team B, one of the two teams will execute complex k_2. If the ball does not pass to the opposite side of the court (e.g., due to a successful block), team B will execute complex k_2; otherwise, team A will execute it. As mentioned above, the probabilities that the ball will pass to the opposite side or not are c and s, respectively. In Fig. 1, the circles represent situations where team A is in possession of the ball, while the squares represent situations where team B is in

Table 3: Probabilities associated with complexes k_1 and k_2.

q_{k_1}	q'_{k_1}	Q_{k_1}	q'_{k_2}	q'_{k_2}	Q_{k_2}
0.50	0.16	0.34	0.42	0.18	0.40

possession. The score at the beginning of the rally is (n, m), where n is the number of points scored by team A, which is serving, and m is the number of points scored by team B. In this way, the rally will evolve through one of the trajectories shown in Fig. 1. As seen in this figure, in this model, the rally is divided into three phases, each corresponding to a specific complex: the serve phase (k_0), the attack phase (k_1), and the counterattack phase (k_2).

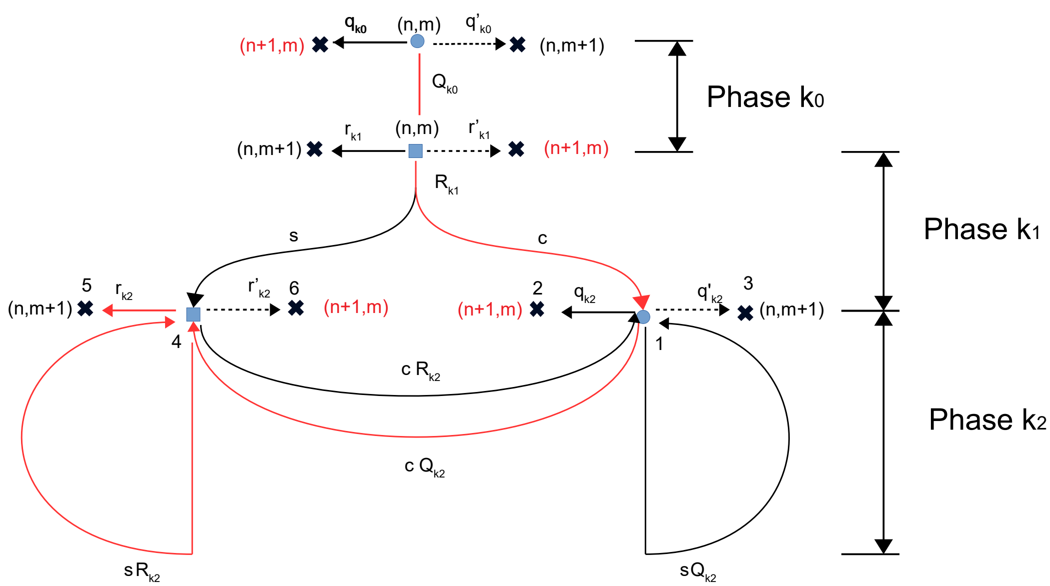


Figure 1: Transition diagram for the probability that team A wins a rally given that it starts serving. The score at the beginning of the rally is (n, m), with n being the points scored by A and m those scored by B. The crosses indicate the states where the rally ends.

States 2, 3, 5, and 6 are absorbing states, meaning that when one of these states is reached, the rally ends. States 1 and 4, on the other hand, represent situations in which the rally continues, with team A and team B executing complex k_2, respectively. To further illustrate the diagram in Fig. 1, consider the trajectory highlighted in red. In this example, team A serves, while team B receives the ball and sets it to the attacker in k_1. The attack is neutralized by team A, which returns the ball in complex k_2 to team B. Team B's counterattack is then neutralized by team A, leaving the ball on team B's side, allowing them to attack for a second time in a row. In this final action, team B wins the rally. The probability P_A^s is the sum of the probabilities of all possible paths that end with team A scoring a point, i.e., with a score of $(n + 1, m)$. Following the procedure described in Appendix A, we obtain

$$P_A^s \;=\; q_{k_0} + Q_{k_0} r'_{k_1} + \frac{Q_{k_0} R_{k_1}}{\mathcal{G}_{k_2}} \left(c\,\mathcal{F}_{k_2} + s\,\mathcal{H}_{k_2}\right), \tag{2}$$

where

$$\mathcal{F}_{k_2} = s\, q_{k_2} R_{k_2} - q_{k_2} - c\, Q_{k_2} r'_{k_2}, \tag{3}$$

with

$$\mathcal{H}_{k_2} = -c\, q_{k_2} R_{k_2} + r'_{k_2} (s\, Q_{k_2} - 1), \tag{4}$$

and

$$\mathcal{G}_{k_2} = c^2 Q_{k_2} R_{k_2} + s\,(Q_{k_2} + R_{k_2} - s\, Q_{k_2} R_{k2}) - 1. \tag{5}$$

The first term of Eq. (2) corresponds to the probability of winning the rally with a serve that directly results in a point (ace). As mentioned above, this value has been documented and is found in Table 2. The second term represents the probability of obtaining the point due to the opposing team making an error while executing complex k_1 (e.g., technical fault, attack error). The last two terms represent the probability of winning the rally during phase k_2.

The probability P_A^r that team A wins the rally given that it starts receiving is defined in a similar way. For this, we take into account that $P_A^r = 1 - P_B^s$, and that P_B^s can be calculated using Eq. (2) under the transformations $q_{k_i} \rightleftharpoons r'_{k_i}$ and $q'_{k_i} \rightleftharpoons r_{k_i}$, thus obtaining

$$P_A^r = r'_{k_0} + q_{k_1} R_{k_0} + \frac{Q_{k_1} R_{k_0}}{\mathcal{G}_{k_2}} \left(c\,\mathcal{L}_{k_2} + s\,\mathcal{M}_{k_2} \right), \tag{6}$$

where it has been defined

$$\mathcal{L}_{k_2} = 1 + c\,q'_{k2} R_{k_2} - s\,Q_{k_2} r_{k_2} + r_{k_2}, \tag{7}$$

and

$$\mathcal{M}_{k_2} = 1 + c\,Q_{k_2} r_{k_2} - s\,q'_{k_2} R_{k_2} + q'_{k2}. \tag{8}$$

The first term represents the probability of winning the rally due to a serve error by the opposing team, while the second term is the probability of winning during the k_1 phase. As before, the last two terms represent the probability of winning the rally during the k_2 phase.

3 Results and Discussion

Since the proposed model depends on several parameters, its validity was assessed by comparing the model results with those reported in the literature, based on extensive statistical observations. Using the values given in Tables 1 and 2 in Eqs. (2) and (6), it is found that when the type of serve is not taken into account, the probabilities of team A scoring a point given that it starts the rally serving and receiving are close to $P_A^s \approx 0.3$ and $P_A^r \approx 0.7$, respectively. This result is in agreement with the values reported in Refs. [15**?** , 28], validating the model's consistency.

From Eq. (2), we calculate the probabilities that team A scores a point in the different phases of the rally for the three types of serves. Using the data from Table 2 as input for the probabilities of k_0 and those from Table 3 for k_1 and k_2, and assuming that performance in those complexes is the same regardless of the type of serve, the results are shown in Table 4. Clearly, for the three types of serves considered, it is more likely to score the point in phase k_2, followed by phase k_1, and with less probability in phase k_0. This result is consistent with empirical observations [29, 30, 31], which may explain why coaches place significant importance on complex k_2 during training. This complex is crucial for retaining the serve in consecutive rallies and is therefore necessary to increase (or reduce) the point difference in a set.

3.1 Theoretical Implications of the Model: The Importance of Useful Serves

Given that the most likely outcome is to reach complex k_2, in addition to trying to score a direct point while serving, the serve must fulfill at least one of the following three objectives: minimize the chances of success for the opposing team in complex k_1 (r_{k_1}); increase the chances that the opposing team will make an unforced error in complex k_1 (r'_{k_1}); or increase the probability of reaching phase k_2 to counterattack (R_{k_1}).

Table 4: Probability of scoring a point in the different phases of the rally given that the team starts serving, with $c = 0.8$.

Serve	Phase k_0	Phase k_1	Phase k_2	P_A^s
JS	0.07	0.11	0.13	0.31
JFS	0.02	0.15	0.17	0.34
FS	0.00	0.16	0.19	0.35

Differentiating by type of serve, but assuming the same performance in complexes k_1 and k_2, it is found that for the JS, $P_A^s \approx 0.31$, while for the JFS and FS, $P_A^s \approx 0.35$. Note that these probabilities have similar values, which is not realistic, suggesting that the FS is the most efficient serve technique. This inconsistency arises mainly because the data used to calculate them do not take into account the effect of useful serves, i.e., they do not include the effect of the type of serve on r_{k_1} and r'_{k_1}. Let $\tilde{r}_{k_1}$, $\tilde{r}'_{k_1}$, and $\tilde{R}_{k_1}$ be the probabilities associated with the rival team's k_1 when the serve is useful, whereas when the serve is neutral, the probabilities are denoted as before: r_{k_1}, r'_{k_1}, and R_{k_1}, and take the values shown in Table 3. It is reasonable to expect that $\tilde{r}'_{k_1} > r'_{k_1}$ and $r_{k_1} > \tilde{r}_{k_1}$, i.e., the receiving team's probability of scoring a point in k_1 is lower in the case of useful serves than in the case of neutral serves. The opposite occurs for r'_{k_1}. Therefore, it is possible to conclude that, although the JS leads to more direct points, it also leads to more serve errors, so that in the end, for the JS, P_A^s can be lower compared to the other types of serves. For the JS to be more efficient than the FS and JFS, it must be aggressive enough that r_{k_1} decreases and r'_{k_1} increases sufficiently to compensate for the serve errors. Clearly, the justification for using the JS lies not only in that it leads to a greater number of direct points but also in its ability to decrease the effectiveness of the rival team's attack in complex k_1; otherwise, it would be less efficient than the other two types of serves considered.

For the JS, the probability that the rally extends beyond the k_0 phase, Q_{k_0}, is close to 0.7, while for the other two types of serves, it is greater than 0.9. Thus, risking the serve with a JS is recommended when the opposing team has a high probability of winning the rally by executing k_1, which is true in elite competitive volleyball but not necessarily in lower categories such as 16U and 14U. This should be considered by coaches when using the serve strategically, as the JFS and even the FS could be useful against teams with a weak attack, i.e., with low r_{k_1} and/or high r'_{k_1} because, in the first case, it is easy to counteract the k_1 of the opposing team, while in the second, it is advantageous to extend the rally to the k_1 phase, where the rival team has a greater chance of making an unforced error.

So far, the probabilities associated with phases k_1 and k_2 have been taken from Table 3. The values reported there do not take into account that they are indirectly affected by the type of serve. To improve the model, it is necessary to consider the effect of the serve type on the probabilities r_{k_1}, r'_{k_1}, and R_{k_1}. Thus, it is necessary to differentiate useful serves from neutral ones since, typically, the JS generates more useful serves than the FS and JFS. This differentiation can be implemented in our model by considering that only a fraction α of the serves decreases the efficiency of the rival team's attack (useful serve), while the rest are perfectly controlled (neutral serve) by the opposing receivers.

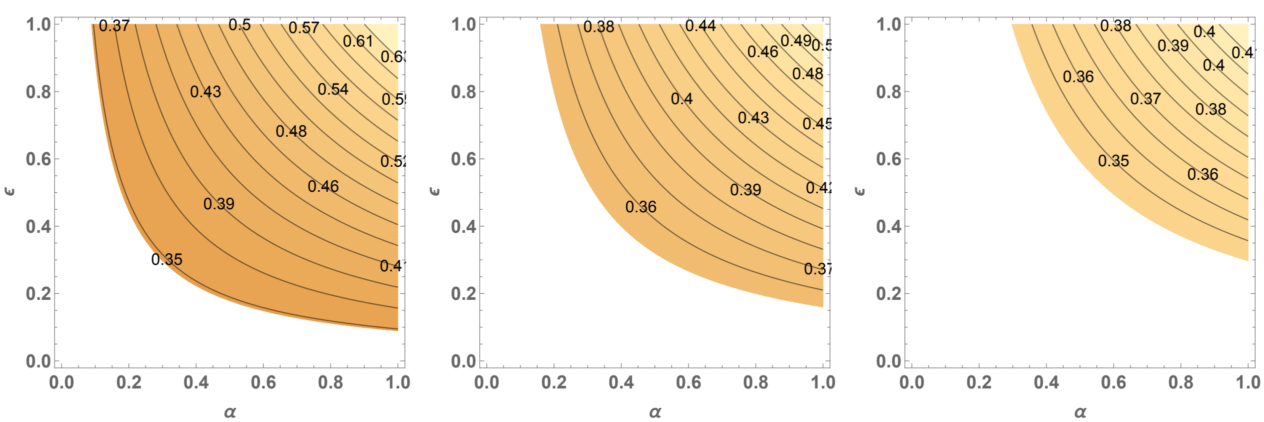
According to Table 1, for the JS, JFS, and FS, we have $\alpha \approx 0.11$, $\alpha \approx 0.05$, and $\alpha \approx 0.03$, respectively. Following a method similar to that used to derive Eq. (2), if neutral and useful serves are distinguished, the probability of winning a rally given that the team starts serving is given by

$$
\begin{aligned}
P_A^s &= q_{k_0} + (1-\alpha)\left(Q_{k_0} r'_{k_1} + \frac{Q_{k_0} R_{k_1}}{\mathcal{G}_{k_2}}\left(c\,\mathcal{F}_{k_2} + s\,\mathcal{H}_{k_2}\right)\right) \\
&+ \alpha\left(Q_{k_0} \tilde{r}'_{k_1} + \frac{Q_{k_0} \tilde{R}_{k_1}}{\mathcal{G}_{k_2}}\left(c\,\mathcal{F}_{k_2} + s\,\mathcal{H}_{k_2}\right)\right),
\end{aligned} \tag{9}
$$

where $\tilde{r}'_{k_1}$, $\tilde{r}_{k_1}$, and $\tilde{R}_{k_1}$ are the analogous probabilities of r'_{k_1}, r_{k_1}, and R_{k_1} for the cases where the serve is useful. Note that in the case of $\alpha = 0$ (no useful serves), Eq. (9) reduces to Eq. (2), as expected.

In the absence of experimental data, we explore below some ways in which the JS could affect the opposing team's chances of a successful attack. For example, consider $\tilde{r}_{k_1} = r_{k_1}(1-\epsilon)$ and $\tilde{r}'_{k_1} = r'_{k_1} + \epsilon\, r_{k_1}$, i.e., the JS decreases the probability that the opposing team scores a point in k_1 by an amount $\epsilon\, r_{k_1}$ and increases by the same amount the probability that the team makes an error in this complex. Hereafter, this scenario will be called Case 1. In Case 1, the probability that the rally reaches phase k_2 remains unchanged, $\tilde{R}_{k_1} = R_{k_1}$. We can also consider cases in which the JS decreases the efficiency of the attack in k_1, according to $\tilde{r}_{k_1} = r_{k_1}(1-\epsilon)$, but this time increasing the probability that the rally continues, $\tilde{R}_{k_1} = R_{k_1} + r_{k_1}\epsilon$, with $\tilde{r}'_{k_1} = r'_{k_1}$ (Case 2). Alternatively, it can also be proposed that the JS increases the probability that the opponent makes an error in complex k_1, $\tilde{r}'_{k_1} = r'_{k_1} + \tilde{R}_{k_1}\epsilon$, with $\tilde{R}_{k_1} = R_{k_1}(1-\epsilon)$ and $\tilde{r}_{k_1} = r_{k_1}$ (Case 3). In all cases, the parameter ϵ satisfies $0 \leq \epsilon \leq 1$.

In Fig. 2, the results found with Eq. (9) for P^s_A as a function of ϵ and α are shown for the three cases mentioned above. The white region represents the set of values of α and ϵ where $P^s_A < 0.35$, i.e., where the JS is less efficient than JFS and FS. Note that Case 1 is more advantageous for team A than the other cases because the region where $P^s_A > 0.35$ is larger. That is, it is more favorable for the JS to increase the opponent's error probability and decrease their effectiveness in k_1 than in the other scenarios considered (Cases 2 and 3). For example, in Case 1, with $\alpha = 0.2$, $\epsilon > 0.45$ is required to achieve $P^s_A > 0.35$, i.e., it is necessary to decrease the opponent's r_{k_1} by 45%. Similarly, for Case 2 and the same value of α, $\epsilon = 0.8$ is required, while for Case 3, it is impossible for the JS to be more efficient than the other types of serves considered. In all cases, the smaller the value of α, the larger the value of ϵ required to satisfy the condition $P^s_A > 0.35$. As mentioned above, although the JS should aim to score more direct serve points compared to the JFS and FS, it is even more important to reduce the opponent's attacking options by decreasing their chance of success in k_1, as exemplified in Case 1. In the proposed model, this effect is represented by the parameters ϵ and α. The challenge here is that the more aggressive the serve, the more likely it is to result in a serve error.



(a) Case 1. (b) Case 2. (c) Case 3.

Figure 2: Effect of the JS on the probability of winning the rally, with $c = 0.8$.

Our model can be applied regardless of whether the team starts the set serving or receiving. Table 5 shows the probability of scoring a point at each phase of the rally given that the team starts the rally receiving. The probabilities were calculated using Eq. (6) and the values reported in Tables 2 and 3. Therefore, the results reported in Table 5 do not include the effect of the type of serve on the rival's performance in k_1. For all three types of serves, it is more likely to score in the k_1 phase, where the probability is approximately three times higher than that of phase k_2. As a result, if a team starts a

Table 5: Probability of scoring a point in the different phases of the rally given that the team starts receiving, with $c = 0.8$.

Serve	Phase k_0	Phase k_1	Phase k_2	P_A^r
JS	0.22	0.36	0.10	0.68
JFS	0.07	0.46	0.13	0.66
FS	0.01	0.50	0.14	0.65

rally performing complex k_1, it is very important to strengthen the serve receive formation to prevent the opposing team from decreasing the efficiency of the team in this complex since, on average, the probability of scoring a point is highest in this phase.

3.2 Practical Implications of the Model: Analysis of the Performance of a Top South American Team in the CSV Men's Tokyo Volleyball Qualification 2020

Below, we illustrate how our model can be used to analyze the performance of a given team. For this purpose, we consider a top South American volleyball team participating in the CSV Men's Tokyo Volleyball Qualification 2020 in Santiago de Chile. Four teams entered the qualifying stage, with each team playing three games. The data presented below were collected from the team's official statistic data scouting using DataVolley 4, which was then double-coded by an independent observer. There was a high level of intercoder agreement. For more information, see Appendix B and the supplementary material. As shown in the supplementary material (SM), the convention used to collect the results of the serve is based on the six items described below:

- Error (=): The ball fails to go over the net, goes out of bounds, or the server is called for a foot fault or time violation.
- Negative (−): The opposing team receives the ball and can attack in any possible way.
- Exclamation (!): The opposing team receives with difficulty, such that the setter has only one attack variant available.
- Positive (+): The opposing team receives the ball such that the setter can choose more than one attack variant, but not all attack options are available.
- Very positive (/): The opposing team's reception is poor, and the ball is sent directly to the other side of the court or cannot be attacked.
- Direct point (#): The opposing team cannot receive the ball, or the serve determines the end of the rally (ace).

Table 6 shows the statistics of the team under study when executing complex k_0.

Table 6: Serve performance of the analyzed team.

Results	=	/	−	!	+	#	Total
Service	45	10	132	32	47	8	274

It is important to note that in the cases of = and #, the rally ends; in the first case, with a point for the team that receives, and in the second case, with a point for the team that serves. In the remaining cases, the rally continues. Furthermore, in this convention, the case $-$ corresponds to the neutral actions mentioned in the previous section, while +, !, and / correspond to useful serves. Note that although the FIVB report uses six categories, we map the results onto the four categories discussed in Sec. 2. Table 6 shows the results for the serves of the team under study. From this data, we can estimate the probabilities associated with complex k_0: $Q_{k_0} = (10{+}132{+}32{+}47)/274 = 221/274 \approx 0.81$, $q_{k_0} = 8/274 \approx 0.03$, and $q'_{k_0} = 45/274 \approx 0.16$.

Similarly, the performance evaluation in complex k_1 is also based on a six-item system as follows:

- Error (=): The ball fails to go over the net, goes out of bounds, or a player is called for a technical violation (e.g., the attacker is called for a net or center line violation during the attack attempt).
- Blocked attack (/): The opposing team blocks the ball, scoring a point.
- Poor ($-$): The ball is easily controlled by the opposing team, allowing them to counterattack.
- Positive (+): The opposing team defends with difficulty, allowing the team that originally attacked to play the ball again.
- Cover (!): The attack is blocked back onto the attacker's side, but a member of the same team digs the ball and the rally continues.
- Direct point (#): The team wins the rally.

The data collected for complex k_1 of the analyzed team is shown in Table 7.

Table 7: Performance of the team under study in complex k_1.

Result	=	/	$-$	!	+	#	Total
k_1	14	12	42	13	11	105	197

Therefore, the probability Q_{k_1} associated with complex k_1 is $Q_{k_1} = (42+11+13)/197 = 66/197 \approx 0.34$. The rally ends with a point for the receiving team at $q_{k_1} = 105/197 \approx 0.53$ and at $q'_{k_1} = (14+12)/197 \approx 0.13$. In cases where the rally continues ("$-$", "+", and "!"), the ball changes possession 80% of the time. In this way, $c \approx 0.8$ and $s \approx 0.2$, as stated before.

The performance of opposing teams in k_1 is presented in Table 8. The numerical data imply $R_{k_1} = (39+11+9)/204 \approx 0.29$, $r_{k_1} = 104/204 \approx 0.51$, and $r'_{k_1} = (22+19)/204 \approx 0.20$. It is worth noting that from this data set, it is obtained $c \approx 0.81$ and $s \approx 0.19$, which reasonably suggests that c and s are independent of the team executing k_1. By comparing the values of q_{k_1} with r_{k_1}, we can also conclude that the performance of the analyzed team in k_1 is similar to that of its opponents.

Table 8: Performance of the opposing team in complex k_1.

Result	=	/	$-$	!	+	#	Total
k_1	22	19	39	11	9	104	204

Analogously, for complex k_2, considering the actions where the attack in k_2 leads to a direct point, it is found that $q_{k_2} \approx 0.46$. On the other hand, attack errors and blocked attacks lead to $q'_{k_2} \approx 0.21$. Finally, the actions where the attack is totally or partially controlled by the opponent lead to $Q_{k_2} \approx 0.33$. The probabilities of the opposing teams in complex k_2 are calculated in the same way, where we get $r_{k_2} \approx 0.47$, $r'_{k_2} \approx 0.23$, and $R_{k_2} \approx 0.30$.

The performance at serve of some of the players from the team under study is shown in Table 9. This table also includes the performance of the respective rotations in complex k_2 and the opposing team's performance in complex k_1. The fraction of the times in which the service has decreased the opponent's attack (α) includes situations (!, +, and /). The five players considered are labeled by the number used on their jerseys: $P1$ (opposite), $P8$ (middle blocker), $P16$ (middle blocker), $P17$ (setter), and $P18$ (right-side hitter). These players were selected because they performed most of the serves in the tournament. Note that only $P1$ executes JS; the other players use FJS.

Table 9 shows that player $P1$ has the most aggressive serve not only because more than 50% of the serves are useful, but also because these serves have a non-zero probability of scoring a direct point. In fact, the smallest probability for the opposing team to score a point in k_1 is found when $P1$ is serving. However, this is the player who commits the most serve errors, and, because of this, as shown below, this rotation does not have the highest winning probability at serve. In contrast, $P16$ has the least aggressive serve, with around 20% of the serves being useful and a negligible probability of scoring a direct point; nevertheless, this is one of the most efficient rotations of the analyzed team. Finally, for the rotation where $P17$ is serving, the opposing team has the highest probability of scoring in k_1 ($r_{k_1} = 0.63$), although a good percentage of their serves are useful.

From Table 8, the average error percentage (=) of the opponents when executing complex k_1 is 11%, while the probability of losing the rally due to the team's block (/) is 9%. In total, the team under study has a 20% chance of winning a rally when it starts serving, given that the opponent executes an attack in k_1. However, from Table 9, the player who scores the most aces while serving, $P1$, only has a probability of 8% of making an ace and a probability of 29% of making a serve error, giving the point to the opposing team. It is then more likely to win the rally by blocking the rival or by an opponent's unforced error in k_1 than by scoring an ace.

Clearly, the performance of the rotations in which these players are serving is not homogeneous. However, the useful serves of $P8$ and $P16$ decrease the opposing team's success probabilities from (r_{k_1}) 0.62 and 0.61 to ($\tilde{r}'_{k_1}$) 0.15 and 0.14, respectively. This represents a decrease of nearly 75% in the rival's performance in k_1. It is worth highlighting that, unlike the case of player $P1$, these players decrease the efficiency of the opposing team with a serve error probability of around 10%, well below the error percentage of $P1$, which is close to 30%.

Table 9: Probabilistic description at serve of some representative players from the analyzed team and their respective rotations.

Probability	$P1$	$P8$	$P16$	$P17$	$P18$
q_{k_0}	0.08	0.00	0.00	0.00	0.00
q'_{k_0}	0.29	0.11	0.10	0.04	0.20
q_{k_2}	0.33	0.40	0.75	0.13	0.75
q'_{k_2}	0.67	0.60	0.25	0.87	0.25
r_{k_1}	0.40	0.62	0.61	0.63	0.54
r'_{k_1}	0.40	0.24	0.19	0.13	0.29
$\tilde{r}_{k_1}$	0.29	0.15	0.14	0.33	0.00
$\tilde{r}'_{k_1}$	0.21	0.31	0.57	0.22	0.22
α	0.58	0.38	0.18	0.38	0.24
Serve type	JS	FJS	FJS	FJS	FJS

The probability of winning a rally when five players from the team under study are serving is calculated using (9) and Table 9. The results are shown in Table 10. The probability of winning the rally in phases k_0, k_1, and k_2 is also included. Although almost 40% of the serves of $P17$ are useful,

Table 10: Probability of winning a rally given that the analyzed team starts the rally serving for five different players.

Player	$P1$	$P8$	$P16$	$P17$	$P18$
P_A^s Eq. (9)	0.34	0.34	0.38	0.20	0.41
Phase k_0	0.08	0.00	0.00	0.00	0.00
Phase k_1	0.18	0.24	0.24	0.16	0.22
Phase k_2	0.08	0.10	0.14	0.04	0.19

the worst performance is found for this rotation, with a winning probability of 20%. This is not only because the serve does not sufficiently decrease the opposing team's chances in k_1 ($\tilde{r}_{k_1} = 0.33$ and $r_{k_1} = 0.63$), but also because the team under study has poor performance in k_2 in this rotation, with a success probability of 0.04%, as shown in Table 10. Note that this rotation has the lowest probability of winning in phases k_1 and k_2. Clearly, this rotation has a poor performance in blocking and setting a counterattack. On the other hand, although $P1$ is the only player who scored direct serve points with a probability of winning in phase k_0 of 0.08, and the only player relying on JS, P_A^s for this player is not the highest in the team under study. This is mainly due to the poor performance of this rotation in k_2, where the probability of winning is only 0.08, but also due to a large number of serving errors. In this case, the benefit of risking the serve is not compensated, and therefore the rotation has a low performance. Furthermore, this rotation has a probability of winning the rally in k_1 of 0.18, which is the second lowest of those that we consider. In contrast, rotations where $P16$ and $P18$ serve have the highest P_A^s, which can be explained not only by the team's good performance in k_2 but also by the impact of these players' useful FJS on the opposing team's performance in k_1. These rotations have the highest probabilities of winning in the k_1 and k_2 phases, see Table 10. The first implies good performance in blocking the rival's k_1 attacks, while the second implies good performance in counterattack.

As mentioned earlier, an important feature of our model is that it allows us to predict the impact of a given increase in performance on the probability of winning a rally. For instance, according to Eq. (9), if $P1$ reduces the number of serve errors in such a way that q'_{k_0} decreases from 0.29 to 0.15, P_A^s increases from 0.34 to 0.41. Our model suggests that decreasing the serve errors of $P1$ would increase the increase the team's probability of scoring a point in that rotation by 20%. The most problematic rotation is found when $P17$ is at serve. However, if this rotation increases its performance in k_2 in such a way that it matches the performance of $P16$ and $P18$ ($q_{k_2} = 0.75$ and $q'_{k_2} = 0.25$), then P_A^s would be 0.39, which corresponds to an increase of nearly 100

Surprisingly, the rotation where the setter ($P17$) is at serve and the opposite is at position four has the worst performance, $P_A^s \approx 0.2$, which is close to half of the best one. In modern volleyball, the winning probability for that rotation is expected to have the largest value because it is designed to have the strongest attack line. Yet, Table 9 shows that when $P17$ is serving, the performance of the team in k_2 is relatively low. Reviewing the videos of the matches, we confirm that this issue is mainly due to inefficient blocking. Block performance in k_2 is related to service performance since a serve that decreases the number of attacking variants of the opposing team facilitates the work of the blockers. In other words, for a given rotation, it is expected that the larger the α, the larger q_{k_1} and q_{k_2}. This is not the case for the analyzed team, where the largest P_A^s is found for the rotation with the smallest α. Although about 40% of the serves of $P17$ are useful, the team cannot take advantage of this due to ineffective blocking. Another reason for the team's poor performance when $P17$ is serving is the limited number of attack variants. The players of the team under study have difficulty scoring a point when the setter has few attack options in k_2 (e.g., when setting the ball to the outside hitter is the only available option). In contrast, the rotations where $P16$ and $P18$ are serving have excellent performance in k_2 despite having a small α. Finally, it is worth mentioning that $P17$ is the player who serves when the analyzed team starts the set serving. According to the results shown in Ref. [10], this decreases the probability of winning the set because this rotation has the lowest P_A^s.

4 Conclusions

Previous work has used Markovian methods to calculate the winning probability of a set in a volleyball match [10]. The present model advances existing approaches by estimating the effect of individual player actions during the different game complexes on the probability of winning a rally. Equation (9) can be used to calculate the chances of winning a rally for particular rotations and players, using as input the probabilities of success in each complex. These probabilities, in turn, measure the team's performance in specific actions such as service, reception, attack, and more. The proposed model allows for determining when a player serving has a better performance than another, not only by the number of direct points they score (ξ) and the number of errors they commit (η) but also by the impact of such actions on the probability of winning the rally. The player who makes the most points or commits the fewest errors while serving is not always the one with the best service performance, as shown by $P1$ in Table 9. Tactical decisions related to the choice of serve technique must consider the serve's impact on the performance of the opposing team's k_1 phase to increase the probability of success in the k_2 phase.

Our model provides a way not only to evaluate the team's performance but also to quantify the effect of non-scoring actions, such as useful serves, on the probability of winning a rally. In other models, such as the one given by Eq. (1), the importance of these actions is evaluated subjectively by assigning an arbitrary weight. The proposed model, however, can predict how much improvement in the performance of an action or set of actions increases the probability P_A^s. Overall, we believe that this type of model could be used by statisticians of high-performance teams to diagnose failures and deficiencies during the different complexes. As in the case of the team under study, the results of the model can be used not only to plan future training sessions but also to correct learning errors in lower categories. For instance, we found that $P1$ has the most offensive serve, i.e., the one with the largest α. However, $P1$'s service has low efficiency due to the number of serve errors. This issue is reflected in the probability P_A^s. For $P1$, we found that $P_A^s = 0.34$, while for players $P16$ and $P18$, these values are 0.38 and 0.41, respectively (see Table 10). However, our model predicts that if $P1$ reduces serve errors by half, P_A^s for that rotation will match the performance of the rotation where $P18$ is serving. A reduction in $P1$'s serve errors will then significantly increase the performance of that rotation.

Admittedly, similar approaches, such as Markovian decision methods, have been used to analyze related sports like beach volleyball. However, we believe that beach volleyball is more tractable as a decision problem than indoor volleyball. Since there are only two players on each side, the number of game combinations is smaller. For example, since only one player blocks, the other must receive. Once a player receives, only one can set the ball during the second contact while their partner is the only attack variant available during the third. This makes strategic decisions more prominent in beach volleyball than in indoor volleyball. Indoor volleyball is primarily a game of power. Heavier indoor volleyballs move quicker and can be hit harder. Beach volleyballs are softer, lighter, and slightly larger than indoor balls. The lighter weight allows them to float more in the air, enabling good players to use the weather to their advantage. When serving, indoor players may have a good idea of which zone they want to aim for, but there is no guarantee who will receive the ball, as a player can easily step out of the reception line while another can drop back to receive. In contrast, common wisdom in beach volleyball dictates serving to the side of the court covered by the weaker, less effective attacker unless the players' serve reception efficiency on the other side of the court is significantly disproportionate; if both players have about the same attack efficiency, players should aim for the weaker setter of the two.

Finally, we note again that the data used in this paper were collected from the team's official statistic data scouting using DataVolley 4, a widely used tool for professional volleyball statistics analysis. The data was double-coded by an independent observer through direct video analysis of the matches, with little disagreement between observers. Since we use a maximum likelihood method for parameter estimation, our model depends on a relatively large number of observations for each category of events. In this respect, our modeling approach has been rather conservative since, although all volleyball rallies involve service actions, not all of them reach k_1 or involve attack actions. Although our model can easily be extended to analyze other game actions, we believe that a model that is reducible to an exact equation and implementable through standardized data collection tools may be more useful as a first step for volleyball players, coaches, and other members of the coaching staff. Data collected from a relatively stable roster of players throughout an entire competition season should, however, meet the increasing demands of parameter estimation. Similarly, it would be useful to collect data during training sessions to obtain baseline estimates of players' performance. From a modeling perspective, this would allow for the implementation of more complex models that consider the relative influence of actions such as useful and non-useful attacks. Naturally, the more categories introduced into the model, the more demanding it will be to collect relevant data that provide stable estimates. However, we do not see this as an intrinsic limitation of the model.

5 Contributions

Contributed to conception and design: DLG, IG-C, DDH Contributed to acquisition of data: DDH Contributed to analysis and interpretation of data: DLG, IG-C, DDH Drafted and/or revised the article: DLG, IG-C Approved the submitted version for publication: DLG, IG-C, DDH

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8 Appendix A

By definition, the probability of scoring a direct service point is q_{k_0}, while the probability of scoring in k_1 is the product $Q_{k_0} r'_{k_1}$. The evolution of the probability of states 1 to 6 in phase k_2 in terms of the number of cycles, ℓ, is given by

$$\vec{p}^{\,\beta}_{\ell} = \mathcal{M}^{\ell} \cdot \vec{p}^{\,\beta}_{0}$$

where

$$\mathcal{M} = \begin{pmatrix} s\,Q_{k_2} & 0 & 0 & c\,R_{k_2} & 0 & 0 \\ q_{k_2} & 1 & 0 & 0 & 0 & 0 \\ q'_{k_2} & 0 & 1 & 0 & 0 & 0 \\ c\,Q_{k_2} & 0 & 0 & s\,R_{k_2} & 0 & 0 \\ 0 & 0 & 0 & r_{k_2} & 1 & 0 \\ 0 & 0 & 0 & r'_{k_2} & 0 & 1 \end{pmatrix},$$

with $\vec{p}_0^{\beta}$ being the probability of the initial state and $\beta = A$ or B. In the case where team A starts serving, $\vec{p}_0^{A} = (1,0,0,0,0,0)$. Let $p_2^{\beta}(\ell)$ and $p_6^{\beta}(\ell)$ be the second and sixth components of the vector $\vec{p}^{\beta}(\ell)$. Then, the probability that team A wins the rally is given by

$$P_A^s = q_{k_0} + Q_{k_0} r'_{k_1} + Q_{k_0} R_{k_1} \lim_{\ell\to\infty} \left(c\,\mathcal{P}^A(\ell) + s\,\mathcal{P}^B(\ell)\right)$$

where $\mathcal{P}^A(\ell) = p_2^A(\ell) + p_6^A(\ell)$ and $\mathcal{P}^B(\ell) = p_2^B(\ell) + p_6^B(\ell)$. Equation (2) is obtained by explicitly taking the limit $\ell \to \infty$.

9 Appendix B

Some of the parameters used in our model can be obtained from the anonymized DataVolley data sheets available in the Supplementary Material (SM). The first twenty-two pages of the SM show the rally-by-rally report of each match, including information such as the player serving, the setter's position, and the action that ends the rally. For example, in the first rally of the final match in the qualification tournament, the setter of the analyzed team ($P17$) is serving while the opposing team's setter is in position 6. The rally ended with an attacking error by $P1$. From page 22 onward, the SM includes the detailed action log of the matches written in standard DataVolley codification. Each action in the different rallies is described, including the player who performed the action, the result of the action, and more. We implemented a Python script to extract the relevant information. We double-checked this data by analyzing the match videos rally by rally. In this way, we used two different and independent observers: a professional volleyball data analyst who was responsible for the report presented in the SM, and two of the authors who analyzed the official match videos. The discrepancies between the two observations were always less than 5%.

One of the quantities not available in the DataVolley reports is the number of useful serves. In this case, for each player, the value of α was calculated as follows. Let $\mathcal{N}$ be the number of serves by a given player where the reception by the opposing team is outside zone three, reducing their offensive options. The parameter α is the ratio between $\mathcal{N}$ and the total number of serves by the player.

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