The mathematical modeling of lactate curves from graded incremental exercise tests

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ABSTRACT

Purpose: Measuring the blood lactate concentration allows for a glimpse at the metabolic processes during exercise. To extract characteristics of metabolism the relationship between blood lactate concentration and power or velocity is modeled. Current modeling approaches allow only limited interpretation, are in conflict with basic principles of scientific mathematical modeling, and lack a phenomenological reasoning.

Methods: We developed a simple analytical expression to model lactate concentration data from graded incremental exercise tests. We compared our new approach to a traditional one in a dataset of N = 24 exercise tests performed by elite junior triathletes.

Results: The new procedure leads to three independent fitting parameters characterizing the baseline lactate concentration, the intensity (power, velocity) at the onset as well as the rate of increase of the lactate concentration. These parameters have a clear meaning and can directly

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be used for diagnostics. They can be interpreted with more confidence compared to the characteristics extracted in the traditional approach.
Conclusion: The performance indicators, naturally appearing in our modeling, should supersede the single points obtained from the traditional evaluation of graded incremental exercise tests ("lactate thresholds"), which can hardly be justified based on the principles of scientific mathematical modeling.

Keywords: lactate testing, anaerobic threshold, endurance performance, performance diagnostics, cycling

Abbreviations

a	fitting parameter of the traditional modeling approach (in mmol/l)
b	fitting parameter of the traditional modeling approach (in mmol/l)
c	fitting parameter of the traditional modeling approach (in W)
c_{La}	blood lactate concentration (in mmol/l)
$c_{\mathrm{La}}(P)$	measured blood lactate concentration as a function of power (in mmol/l)
$c_{\mathrm{La}}^{\mathrm{rest}}$	lactate rest concentration (in mmol/l), as a fitting parameter of the novel modeling approach
N	number of participants
P	power (in W)
P_1	low power scale characterizing the first onset of the increase of the lactate concentration (in W), as a fitting parameter of the novel modeling approach
P_2	high power scale characterizing the exponential growth of the lactate concentration (in W), as a fitting parameter of the novel modeling approach
P_4	power corresponding to a lactate concentration of 4 mmol/l (in W)
SSE	sum of squared differences

INTRODUCTION

Measuring the blood lactate concentration allows for a glimpse at the processes usually hidden inside a human's body. Various tissues accumulate, transport, and eliminate lactate; all these processes contribute to the measured blood lactate concentration (van Hall 2010). Researchers and practitioners have investigated blood lactate concentrations for the past decades in the fields of biology, medicine, and sports.

When individuals master a workload of increasing intensity (e.g., in graded incremental exercise tests), the amplified substrate-level phosphorylation leads to a rising blood lactate concentration. The evaluation of the resulting lactate-power or lactate-velocity relationship allows for an estimation of the physical fitness, as it reflects the net metabolic processes during exercise. To be efficient, we from now use the word power only, when referring to a work intensity, instead of both power and velocity.

Usually, researchers and practitioners only locate single characteristic points of the dependence of the lactate concentration on power $c_{\mathrm{La}}(P)$. Sports scientists call these points "lactate thresholds" (Faude et al. 2009). Often the workload at a fixed blood lactate concentration (e.g., $c_{\mathrm{La}}(P)$ = 4 mmol/l) during a graded incremental exercise test is interpreted as an approximation of the power that corresponds to a critical condition in metabolism (the maximal lactate steady-state, MLSS). But the exact definitions of these "thresholds" seem to lack physiological reasoning. Furthermore, the lactate-power relationship provides more information than obtained by extracting single points.

To extract general characteristics of lactate curves, we need to model the lactate-power relationship (Beaver et al. 1985, Hughson et al. 1987, Morton 1989, Lundberg et al. 1986, Morton et al. 1994, Newell et al. 2006, Bentley et al. 2007). A proper model translates the assumed and observed behavior of lactate curves into mathematics. Current applied modeling approaches in exercise science do not capture all trends of the lactate-power relationship. The parameters of the model should be meaningful and easy to interpret. For lactate curves from incremental exercise tests, three main characteristics exist: the baseline concentration, the onset of the rise, and the rate of increase of the curve. Adequate modeling should allow us to separately quantify these parameters. The comparison of the parameters between athletes and in the longitudinal direction may help to provide information on differences and changes in the physiological

processes involving lactate. These insights may improve our understanding of the mechanisms of individual training response and of the exercise performance development.

In this paper we develop a new approach to model the behavior of the lactate concentration in graded incremental exercise tests. We demonstrate that our modeling is accurate and allows to separately quantify the baseline, the onset, and the rate of increase of the lactate curve. The fitted parameters have standard deviations that are significantly smaller than the ones obtained in alternative approaches. In this sense our model is more "reasonable" and "stable". Crucially, our approach obeys the basic principles of scientific mathematical modeling.

MODELLING

Data Set

We analyzed a data set of N = 24 graded incremental exercise tests performed by junior triathletes of national level (13 males, 11 females, age: 14 to 19). The data were collected by the German Research Centre of Elite Sport as part of the momentum project of the German Sport University Cologne. Athletes performed the test on a stationary cycling ergometer (SRM, Schoberer Radmesstechnik, Jülich, Germany). The initial load was 40-100 W (based on age, sex, and bodyweight) and increased by 30 W every 3 minutes until exhaustion. If athletes did not fully complete their last power step, we linearly interpolated the power according to the fraction of time they persevered. Capillary blood samples were taken from the earlobe at the end of each exercise step and analyzed using an enzymatic-amperometric sensor chip system (Biosen C-Line, EKF-diagnostic GmbH, Barleben, Germany).

The traditional fitting procedure

For lactate data from graded incremental exercise tests, various forms of fitting procedures exist in the literature (Beaver et al. 1985, Hughson et al. 1987, Morton 1989, Lundberg et al. 1986, Morton et al. 1994, Newell et al. 2006, Bentley et al. 2007). We here utilized a reference function, that stands as an often used example for these traditional models (Hughson et al. 1987), namely

$$f_{\mathrm{ref}}(P) = a + b\exp(P/c) \tag{1}$$

with the three fitting parameters a, b, both of dimension concentration, and c of dimension power. The two basic assumptions leading to this function are the idea of a rest lactate level $a + b$ (at $P = 0$) and an exponential growth with rate $1/c$ with increasing exercise intensity which starts at $P = 0$. Further down we comment on other fitting forms assumed in the past. With

$f_{\mathrm{ref}}(P)$ determined, it is possible to compute the power P_4 at exactly 4 mmol/l of blood lactate concentration (Mader et al. 1976, Sjödin and Jacobs 1981). Basic algebra leads to

$$P_4 = c \ln\left(\frac{4-a}{b}\right) \tag{2}$$

with a and b given in mmol/l. This type of interpolation is frequently used in the software packages of diagnostic laboratories.

The new fitting procedure

We used a new mathematical model to fit the lactate-power relationship. Our model is based on the following commonly accepted observations and phenomenological ideas on how the lactate concentration measured in graded incremental exercise tests depends on power:

(i) For an extended interval at the lower end of powers the lactate concentration barely rises above its rest value $c_{\mathrm{La}}^{\mathrm{rest}}$. In fact, the lactate concentration often decreases first, and starts to increase only for larger power (Newell et al. 2006). In any case, all changes in blood lactate concentration observed in the low-power regime are much smaller than the ones observed at larger power (see Fig. 1).

(ii) From a first characteristic power scale P_1 on, c_{La} rises exponentially with P. The phrase "exponential growth" is often used synonymously with "a strong dependence on the variable". It, however, means much more. In the language of the present context, the characterizing property of exponential growth is, that the change of the lactate concentration with power (the derivative) is proportional to the current lactate concentration (at power P).

(iii) The exponential growth for $P > P_1$ naturally sets a second characteristic power scale P_2. The argument of the exponential function has to be dimensionless, and one has to divide P by a scale P_2 of dimension power before inserting it as the argument. The appearing term is thus of the form $\exp(P/P_2)$. The larger P_2 the slower the exponential function grows with increasing P; in fact, $1/P_2$ is a measure for the growth rate.

In the mathematical modeling on this phenomenological level, we treat both power scales as independent. From a practical point of view "independence" means that both can be addressed separately in properly designed training schemes. We finally assume that the fitting function is continuous (no jumps for varying P) which appears to be natural. Then (i) to (iii) lead to the simple and transparent new fitting function

$$f_{\mathrm{new}}(P) = \begin{cases} c_{\mathrm{La}}^{\mathrm{rest}} & , \quad \text{for } P \leq P_1 \\ c_{\mathrm{La}}^{\mathrm{rest}} \exp[(P - P_1)/P_2] & , \quad \text{for } P > P_1 \end{cases} \tag{3}$$

with the two power scales P_1 and P_2 as well as the rest lactate concentration $c_{\mathrm{La}}^{\mathrm{rest}}$ as fitting parameters. We emphasize that all three have a clear meaning (see Fig. 2). The first derivative (the slope) of the (continuous) fitting function jumps at P_1. This should not be misinterpreted. We do not believe that the underlying physiological processes comprise this discontinuity of the first derivative (Morton 1989). It is merely a consequence of the goal to incorporate the commonly accepted phenomenological ideas (i) to (iii) into a simple and robust mathematical model.

Comparison of models

We fitted both the traditional and our new approach to the $c_{\mathrm{La}}(P)$ data of each graded incremental exercise test in our data set. We used a least-square optimization routine to find the individual fit parameters for each test. We computed the residual for each lactate data point, i.e., the difference between the concentration measured and the concentration obtained by the fit. As a measure for the fit quality, we computed SSE as the sum of squared residuals and the relative standard deviation of the fitting parameters. For simplicity the unavoidable error of the lactate concentration, which can only be measured up to a certain precision, as well as the uncertainty of the power were neglected. Concerning the former this implies that the standard deviation of c_{La} in the computation of SSE to be minimized in the least square fit was set to one. On the one hand, SSE provides an estimate how well the desired model fits to the data points. On the other hand, the relative standard deviation of the fitting parameters reveals how robust the parameters are towards changes. A large relative standard deviation thus indicates that the corresponding parameter is not very reliable given the model and the measured data. Large relative standard deviations can appear despite a reasonable SSE and provide another measure for how reasonable a fitted line shape is. We also refer to this as the stability of the fit. A small relative standard deviation, in particular, indicates that we can be confident when interpreting the fitting parameters. We compared both measures for the fitting quality between the two models. We used Python version 3.9.0 and R version 4.1.1 for data analysis and visualization. All raw data and scripts are available at https://osf.io/av6gm/.

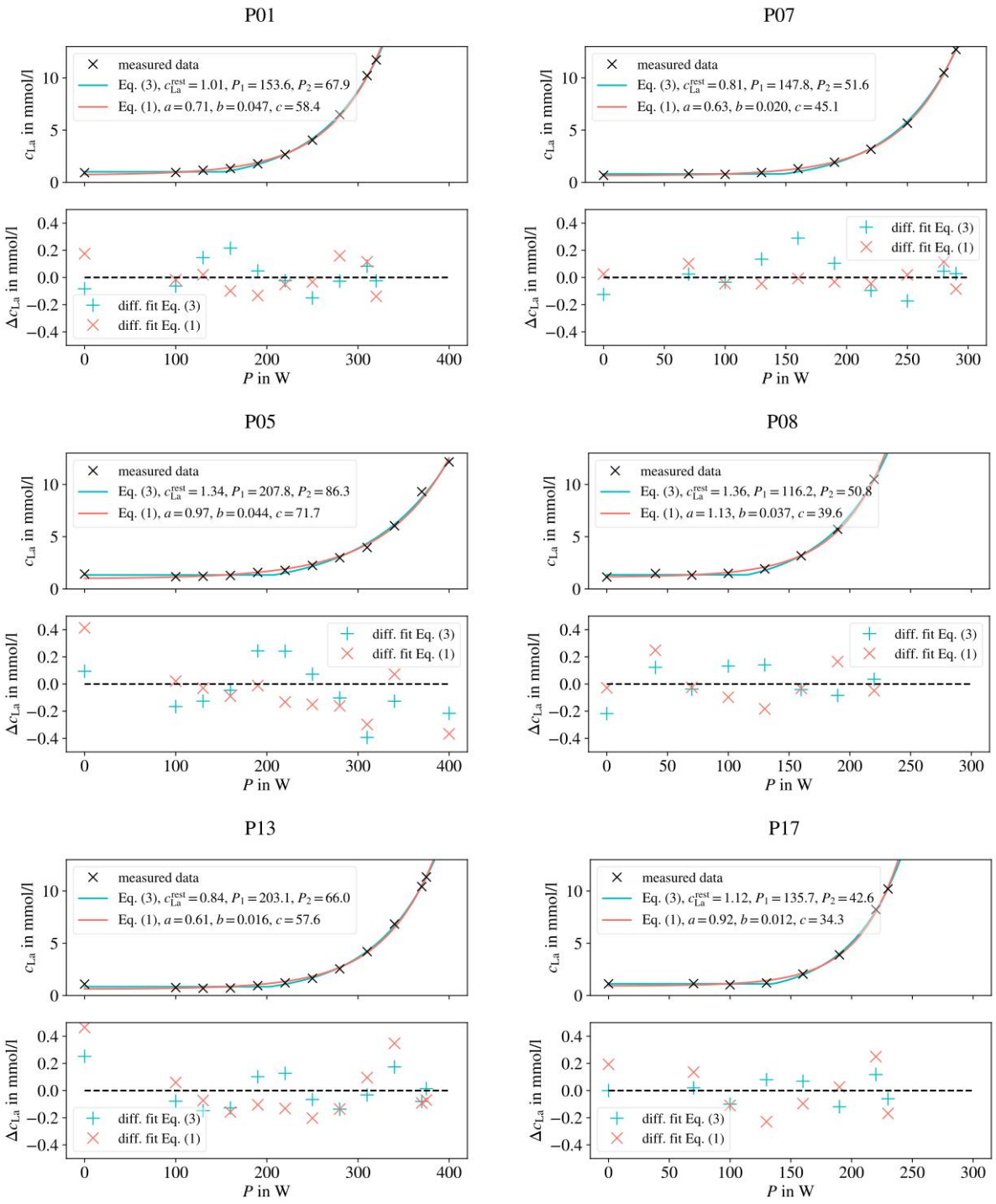

Figure 1. Application of two different fitting procedures on data from six graded incremental exercise cycling tests. Black crosses in the upper panels: Measured data for the lactate concentration c_{La} as a function of the power P for six elite junior triathletes; three females (P01, P05, P13) and three males (P07, P08, P17). The green lines show a fit of the newly suggested Eq. (3) to the data and the red ones of the traditional Eq. (1). The fitting parameters are given in the legends. c_{La}^{rest}, a, and b are given in unit mmol/l and P_1, P_2, and c in unit W. Lower panels: The corresponding residuals Δc_{La} of the data and the fits are shown as the symbols. Note the color coding.

RESULTS

Both models reflect the general trend of a rising lactate concentration with higher power, but the traditional fitting form cannot capture the characteristic of a constant or even declining concentration during the first power steps (see Fig. 1). The new model results in more stable fitting parameters, i.e., smaller relative standard deviations, while maintaining a similar accuracy of fit, i.e., a similar SSE, compared to the traditional model (see Fig. 3).

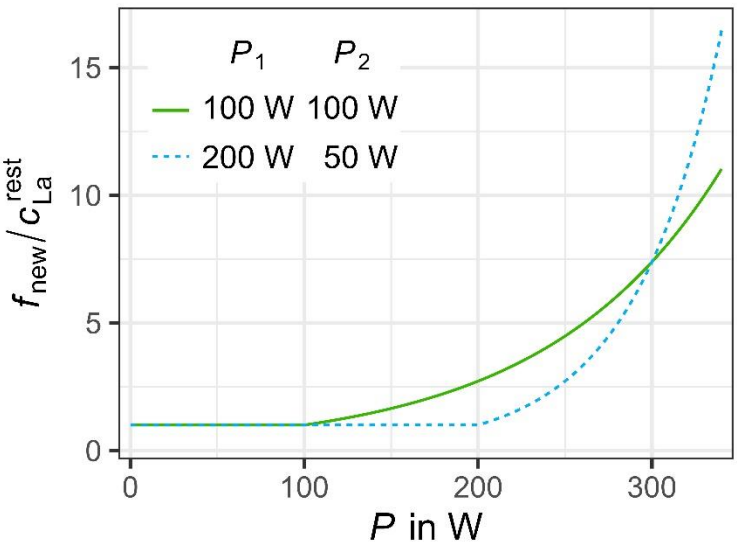

Figure 2. Sketch of the dimensionless new fitting function f_{new}/c_{La}^{rest} as a function of the power P (in W) for two different sets of characteristic power scales (P_1, P_2). P_1 characterizes the onset of the exponential rise. A higher P_1 corresponds to a later onset. P_2 quantifies the rate of the exponential rise. A lower P_2 corresponds to a steeper increase.

DISCUSSION

The new modeling approach allows us to quantify the characteristics of lactate concentration curves from graded incremental exercise tests. It better captures the observed trends in lactate kinetics and leads to more stable fitting parameters than traditional approaches.

The traditional fitting procedure

The traditional modeling of $c_{La}(P)$ yields a continuously increasing function (see red lines in Fig. 1). Most importantly, it assumes $c_{La}(P)$ to be exponential even for low power, which is clearly not the case in reality. In contrast, the measured data shows, that c_{La} only rises exponentially for

higher powers. To still fit the function to the data, it is mathematically necessary that the parameter b is exceedingly small. This leads to an unstable fit, which is demonstrated by the large relative standard deviation of the fitting parameters (see Fig. 3b).

The parameter c is the only characteristic power scale of the traditional modeling function; it is the only parameter of the dimension power. This contrasts with the common belief, that lactate curves from graded incremental exercise tests contain two independent power scales. Even the single power scale c does not provide valuable insights. For example, in Fig. 1 P08 has a higher c than P17, despite a slightly lower maximal power achieved during the test. On the contrary P07 has a higher c with a higher maximal power output. Thus, the traditional fit does not capture the trends of $c_{\mathrm{La}}(P)$; and its parameters are unstable and cannot be interpreted directly.

The new fitting procedure

The new fitting approach yields a constant value of $c_{\mathrm{La}}(P)$ until a first power scale, from which on $c_{\mathrm{La}}(P)$ rises exponentially (see green lines in Fig. 1). This matches the observations of the lactate concentration being constant within the measurement accuracy or even moderately declining during the first steps of the test. The minor variations that occur during this phase do not influence the characteristic value P_2. All fitting parameters can be interpreted with confidence, as their relative standard deviation is comparably small (see Fig. 3). The modeling function has the two independent power scales P_1 and P_2. According to the basic principles of scientific mathematical modeling, the definition of these scales should be based on a reasoning relying on dimensional quantities. It should not rely on fixed dimension-full values, such as, e.g., an intensity at a lactate concentration of 4 mmol/l, but stand in reference to another naturally occurring value, e.g., the rest lactate concentration. For an illustration of this issue think of the concept of half-life in a decay process, which is useful in virtually any science. The half-life is defined as the time at which only half the initial amount remains and thus requires the reference to the initial amount. One first divides the current amount by the initial one, leading to a dimensionless expression, and then asks which value of the variable renders this ratio to be 1/2. In our new approach of modeling lactate curves, we define the characteristic power scales by referencing the data to the rest lactate concentration; we can divide the new fitting equation by $c_{\mathrm{La}}^{\mathrm{rest}}$ to make it dimensionless (see Fig. 2). The first characteristic scale P_1 is set by the smallest power at which $\frac{f_{\mathrm{new}}(P)}{c_{\mathrm{La}}^{\mathrm{rest}}} > 1$, while P_2 is defined by $\frac{f_{\mathrm{new}}(P_1+P_2)}{c_{\mathrm{La}}^{\mathrm{rest}}} = \exp(1)$, i.e., the power at which the

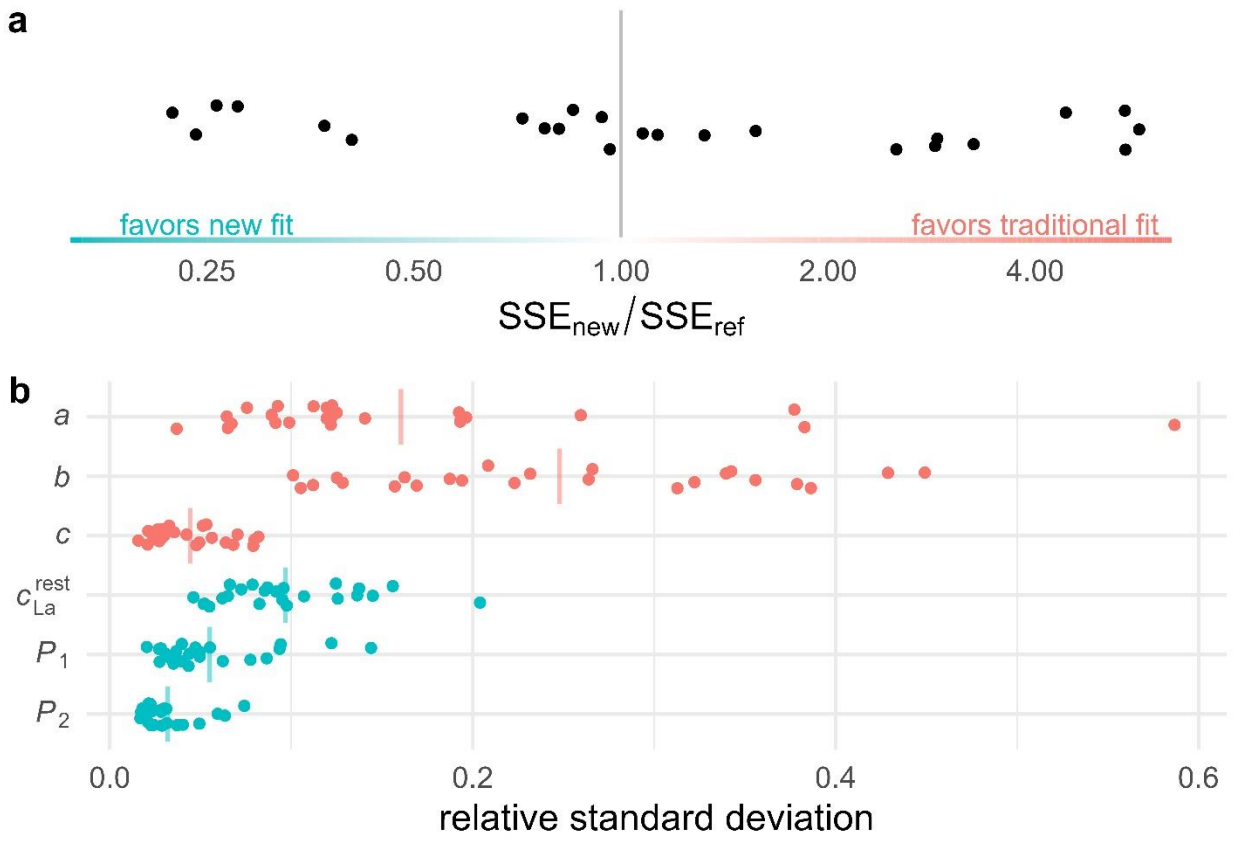

Figure 3. The new approach is more stable than the traditional one by maintaining a similar accuracy. **(a)** Comparison of SSE (sum of squared errors) for the 24 tests of our data set. A low ratio SSE_{new} / SSE_{ref} means that the new fit is more accurate than the traditional, and vice versa. The ratio is displayed on a $\log_{10}$ scale. Data is vertically jittered to improve visibility of the single data points. **(b)** Comparison of the relative standard deviation for the three fitting parameters of the traditional (red) and the new model (green). Lower values indicate a higher certainty when interpreting these parameters. The vertical lines represent the mean values for each parameter.

lactate concentration rises an universal factor $\exp(1)=2.718...$ (Euler's number) above the rest lactate concentration.

Our mathematically concise modeling leads to two independent power scales, which are easy to interpret: P_1 stands for the onset of the exponential rise, and P_2 characterizes the rate of this rise. A lower P_2 means a stronger exponential rise; a higher P_2 leads to a "flatter" curve. Combining both power scales allows us to fully describe and quantify the characteristics of the lactate curve (see Fig. 1). As an example, P05 and P13 both show similar P_1, but for P05 P_2 is higher. This means, that despite a similar power at the onset of the exponential rise, P05 can perform at a higher maximal power, as his exponential rise is less steep. P01 and P13, in contrast,

share a similar P_2 but differ in P_1: Their exponential rise is equally steep, but it starts earlier for P01 in comparison to P13. Accordingly, different combinations of P_1 and P_2 can lead to similar performance levels in athletes. For example, P08 and P17 show an almost similar peak performance originating from different characteristics of $c_{\mathrm{La}}(P)$: for P08 P_1 is low and P_2 high; for P17 the opposite holds. The higher performance level of P07 as compared to P08 and P17 can be mostly explained by a later onset of the exponential rise, which is quantified in the higher value of P_1.

The proposed fitting function allows for new theoretical insights from lactate concentration measured during constant load tests. As we are now able to separately quantify P_1 and P_2, we can speculate about their physiological origins. We assume the low power scale P_1, on the one hand, to be mainly associated with the oxidative phosphorylation. Workloads smaller or equal P_1 lead to metabolic states in which the rate of substrate-level phosphorylation and the rate of lactate removal are in equilibrium, which leads to a constant blood lactate concentration. An increased power of oxidative phosphorylation can both cause a late onset of the rate of substrate-level phosphorylation and a high rate of lactate removal. We assume P_2, on the other hand, to be more strongly associated with the substrate-level phosphorylation itself. A high activation of the substrate-level phosphorylation may lead to a steep rise of the lactate concentration once the offset has been reached; and thus, a low P_2.

The new model also has practical implications for performance diagnostics. Our approach may reveal different physiological profiles of athletes. Based on the determined P_1 and P_2 it might be meaningful to target these parameters with specified training. For example, P13 in Fig. 1 might want to increase his P_2, while the main goal of P01 is to increase his P_1. Which physiological stimuli lead to changes in the characteristic power scales remains a subject of further longitudinal research. Based on the two power scales P_1 and P_2 one can also define training zones. How the zone boundaries should be computed from these two parameters depends on the number of zones one is aiming at. For four zones a simple scheme would be to, e.g., consider the intervals $Z_1 = [P_1/2, P_1]$, $Z_2 = [P_1, P_1 + P_2]$, $Z_3 = [P1 + P_2, P_1 + 2P_2]$, and $Z_4 = [P_1 + 2P_2, P_1 + 3P_2]$. We intentionally avoid denoting them by often used names such as, e.g., active recovery, maximal fat oxdiation, lactate threshold, …. However, taking the sum of other fractions or multiples of P_1 and P_2 to define the zone boundaries is conceivable and might be more appropriate. This fine tuning of training zones built on both characteristic power scales should be a subject of future research, while bearing in mind, that such training zones are influenced by many athlete-specific factors, which cannot be fully assessed by a diagnostician.

Comparison of both approaches

The fitting parameters of our new model can be directly interpreted, which is not possible with the traditional model. The traditional model's power scale c allows only minimal insights when comparing it between athletes. But we can easily interpret the parameters of the new model: A higher P_1 means a later onset of the curve; and a higher P_2 means a less steep rise (see Fig. 2).

The overall quality of the fitting of Eq. (1) and Eq. (3), measured by the SSE is on average the same (see Fig. 3a). A closer look at the lower panels of Fig. 1 reveals that the regime of exponential growth of the blood lactate concentration with power is, however, generically described better by the new fit. In these panels the green plus signs are mostly closer to the dashed zero line than the red crosses. This is not surprising as this fitting form only assumes an exponential growth where it usually really occurs. In accordance with this, the relative standard deviation of the high-power scale P_2 is on average smaller than that of the power scale c (see Fig. 3b); P_2 is thus a more robust measure of the endurance capacity than c.

Taking the line shape of the measured data into account, it is not surprising that P_1 is prone to fluctuations, that is, the relative standard deviation of this fitting parameter of Eq. (3) is larger than that of P_2. Where exactly one has to switch from a constant value to the exponential growth is not very well defined. P_1 still renders to be much more robust than the parameter b (see Fig. 3b). The relative standard deviation of $c_{\mathrm{La}}^{\mathrm{rest}}$ can reach up to 20% but is significantly smaller than that of a. We compare these two as the rest lactate concentration $a + b$ in Eq. (1) is dominated by $a \gg b$. All parameters indicate a more stable fit of the measured data with the new modeling function.

Other fitting forms and mathematical procedures

We next comment on other mathematical procedures performed and fitting forms assumed to evaluate graded incremental exercise tests. In some, instead of the power, the rate of oxygen uptake $\dot{V}\mathrm{O}_2$ (gas exchange) is used as the variable. To a good approximation $\dot{V}\mathrm{O}_2$ and the power P show a linear relationship in the regime of interest to us (Mader and Heck 1991). For the present purpose one can therefore identify the two.

Beaver et al. (1985) suggested to plot the data on a log-log scale to better identify the variable at which c_{La} switches from "… very slow increase …" to "… rapid increase …". The definition of the crossover scale at which the behavior changes and which might roughly correspond to P_1, remains vague and relies on visual inspection. Furthermore, the authors argue in favor of power-law behavior beyond this scale, instead of an exponential one, as in Eq. (3) and the reference fitting form Eq. (1) (Hughson et al. 1987). This ignores that power laws can only be inferred

empirically if data spanning several orders of magnitude exist, which is obviously not the case; compare, e.g., Fig. 1. Power laws have furthermore the property that they are scale-free. If the data would indeed follow a power law, it would not be possible to extract a second characteristic high-power scale. To overcome the problem of visual inspection, Lundberg et al. (1986) proposed to use a computerized approach to identify the crossover point.

The differences between the often used reference fitting form Eq. (1) (Hughson et al. 1987) and the analysis of Beaver et al. (1985), led to a dispute between the two groups in form of a Letter to the Editor and a Reply (Beaver et al. 1988, Hughson and Swanson 1988). Interestingly, this quarrel was exposed as a sham debate by Morton (1989) shortly after. He revealed the mathematical and logical weaknesses of both papers. Unfortunately, his concise analysis, which we fully subscribe to, went mainly unnoticed; while the publications of Beaver et al. (1985) and Hughson et al. (1987) combined were cited roughly 500 times up to today, Richard Hugh Morton's paper was only referred to 17 times (both according to the Web of Science).

Morton et al. (1994) later on suggested a fitting form with three segments, the first being a constant, the second and third each being second order polynomials. The two characteristic (power) scales then correspond to the “breakpoints” at which the three curves are glued together. This leads to a model with five fitting parameters, which, given the typically 5 to 15 data points, appears to be a lot. In any case, this goes beyond the minimal mathematical model we are aiming at and indeed requires justifications which exceed the phenomenological level (Morton et al. 1994).

Newell et al. (2006) put forward the purely statistical approach called functional data analysis to fit and analyze the data. Compared to our new approach this is clearly more complex and, in addition, ignores to a large extend the underlying commonly accepted phenomenological ideas (i) to (iii). The statistical approach is unbiased, which, in turn, makes it difficult to provide answers to the straightforward questions posed by athletes and coaches. In other words, our new approach is guided by mathematical modeling as used in natural sciences, which is always based on model ideas, while that of Newell et al. is purely guided by statistics.

Besides extracting points of as fixed lactate concentration, a number of further procedures to define “lactate thresholds” or to extract them from data measured in graded incremental exercise tests were suggested (Bentley et al. 2007, Stegmann et al. 1981, Orr et al. 1982, Davis et al. 1983, Cheng et al. 1992, Baldari and Guidetti 2000). Some are based on visual inspection of the data; other are computerized. But all of them rely on ad hoc assumptions and their modeling suffers from the shortcomings discussed in this article. The data or the analytical

expressions obtained by fitting the data are not brought into dimensionless form prior to the definition of characteristic scales.

Limitations

The modification of test parameters (step duration, load increase,…) influences the lactate kinetics – and thus the lactate-power relationship – of graded incremental exercise tests (Bentley et al. 2007). This affects not only our new fitting form, but all modeling approaches of such tests. To compare lactate concentrations between and within athletes it is crucial to employ identical protocols or protocols that lead to similar lactate responses. Based on its underlying ideas of a specific lactate behavior, our approach is optimal for, yet limited to, test protocols with an increasing load. Other tests require different modeling approaches to reflect the basic ideas of the metabolism's response to the load. As our new procedure mathematically incorporates the idea of a constant lactate concentration during the first power steps, it requires a low starting power of the test (in relation to an athlete's performance level). This is common for most graded incremental exercise tests in sports science and commercial diagnostics of trained athletes.

SUMMARY AND OUTLOOK

In this paper we suggested a minimal model function to fit the measured blood lactate concentration data of graded incremental exercise tests. The novel approach strictly follows the principles of mathematical modeling in sciences, such as, a proper treatment of dimensions, an appropriate number of fitting parameters, and characteristic scales of the variable which appear naturally on the right-hand side of the equation defining the functional relation. The commonly accepted phenomenological ideas are properly reflected in the fitting function which only has three parameters: the rest lactate concentration, the onset of rise of the blood lactate concentration as well as its rate. All three can directly be used for diagnostics and the two power scales to define training regimes. This leads to a highly transparent and simple evaluation of graded incremental exercise tests.

We are of course aware, that many commercial diagnostic laboratories do not only rely on the measured $c_{\mathrm{La}}(P)$ and a fitting procedure such as Eq. (1) to provide athletes and coaches with indicators for the ability to perform in competitions as well as with training regimes. They use in addition data from gas exchange (if available), simulation tools (Mader and Heck 1986, Hauser et al. 2014), and the insights gained from collecting thousands of $c_{\mathrm{La}}(P)$ data sets over the years (big data approaches). However, a faithful mathematical modeling, as we suggested it, is a basic ingredient of even these more elaborate approaches.

We here focused on cycling tests. Preliminary attempts to analyze running tests in the same way indicate that our novel approach can be equally useful for such.

Finally, we would like to comment on the question if the performance metric $(c_{\mathrm{La}}^{\mathrm{rest}}, P_1, P_2)$ obtained from the above analysis reflects the insights gained from improved measures such as, e.g., the power in the MLSS. To investigate this, one would have to study the correlations between measured values for the power in the MLSS and our characteristic power scales. This was done in the past for the traditional "lactate thresholds" (Heck et al. 1985, Lajoie et al. 2000, van Schuylenbergh et al. 2004, Beneke 1995, Jones and Doust 1998, Hauser et al. 2014) and requires as many data sets as possible. The bottleneck is the time-consuming precise determination of the power in the MLSS, which can only be obtained in a series of (at least) 30 minutes constant load tests. Based on these studies it is commonly believed that the relation between the lactate concentration and the power measured in graded incremental exercise tests can be used to predict the power in the MLSS. Provided this is correct, a unique mathematical function of our three fitting parameters must exit, which provides a reliable estimate for the power in the MLSS. Determining this function remains to be a challenge for the future.

We presented a new modeling approach for the measured blood lactate concentration during graded incremental exercise tests. The physiological background of our modeling parameters and their modification with appropriate training stimuli should be subject of further research. While most of the physiological processes remain in hiding, our approach may sharpen the limited view we have on what happens inside the human's body.

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Contributions

SN, OJQ and VM conceived and designed research. VM analyzed data. SN and VM prepared figures. SN and VM drafted the manuscript. SN, OJQ and VM edited and revised the manuscript. SN, OJQ and VM approved the final version of manuscript.

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Data and Supplementary Material Accessibility

All data, code and figures can be found at the Open Science Framework (https://osf.io/av6gm/)

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